Southwest Fisheries Center Administrative Report No. 12H, 1977

BFISH: A POPULATION DYNAMICS AND FISHERY MANAGEMENT MODEL

William S. Lovejoy
Southwest Fisheries Center
National Marine Fisheries Service, NOAA
Honolulu, HI 96812

June 1977

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I. INTRODUCTION

The BFISH computer model was initiated to lend insight into the possible effects of alternative fishery management schemes on the time dependent population sizes and geographical distributions of a variety of species of fish, and on some return function associated with those populations. BFISH is a multi-compartment, multi-species simulation model incorporating birth, death, and intercompartmental migration for each species. Several alternative management schemes have been internalized, others may be investigated by manipulation of the time dependent input coefficients. The return function presently incorporated is the fishing yield (in numbers of fish).

Specifically addressed are the attributes of the Hawaiian tuna and billfish fishery, but the model is constructed in a general nature, with the user supplying the compartmental structure and species characteristics. Thus, application to alternative management situations should yield excellent results providing the attributes of the problem are compatible with the assumptions made in the BFISH numerical formulation.

II. NUMERICAL FORMULATION

A. Population Dynamics

The geographical and chronological variations in fish populations are numerically modeled by discretizing the populations in space and time. Geographical compartments are set up by the user, and BFISH calculates the populations in each compartment at discrete, user-specified time intervals. For each species, a single age-class model is utilized that incorporates birth and death in each compartment and migration between compartments.

Numerically, the population dynamics problem is formulated as follows:

$$\frac{dP_{ij}(t)}{dt} = -\left(FC_{ij}(t) + FS_{ij}(t) + XN_j + EM_{ij}(t)\right) P_{ij}(t) + SUM_{ij}(t) + RC_{ij}(t) \tag{1}$$

where

$$FC_{ij}(t) = EC_i(t)*QC_{ij}(t)$$

$$FS_{ij}(t) = ES_i(t)*QS_{ij}(t)$$

$$EM_{ij}(t) = \sum_{k \neq i} XM_{i \to k,\, j}(t)$$

$$SUM_{ij}(t) = \sum_{k \neq i} XM_{k \to i,\, j}(t)*P_{kj}(t)$$

$$RC_{ij}(t) = \text{minimum} \begin{cases} FR_{ij}(t)*RP_j(t - m_j)* \sum_k P_{kj}(t - m_j) \\ FR_{ij}(t)*RCMAX_j \end{cases}$$

and

$EC_i(t)$ = commercial fishing effort in compartment i at time t.

$ES_i(t)$ = sport fishing effort in compartment i at time t.

$FC_{ij}(t)$ = commercial fishing mortality coefficient for species j in compartment i at time t.

$FS_{ij}(t)$ = sport fishing mortality coefficient for species j in compartment i at time t.

$P_{ij}(t)$ = population of species j in compartment i at time t.

$QC_{ij}(t)$ = commercial fishing catchability coefficient for species j in compartment i at time t.

$QS_{ij}(t)$ = sport fishing catchability coefficient for species j in compartment i at time t.

$RO_{ij}(t)$ = recruitment rate to species j in compartment i at time t.

$RMAX_j$ = maximum recruitment rate for species j.

$RP_j(t)$ = reproduction coefficient for species j at time t.

$XM_{i \to k, j}(t)$ = migration coefficient for species j migrating from compartment i to compartment k at time t.

XN_j = natural mortality coefficient for species j.

$FR_{ij}(t)$ = fraction of new recruits to species j at time t that will be recruited to compartment i.

m_j = maturation age (age when fish enter the fishery) for species j.

The BFISH model incorporates the concept of a time cycle equal to some integral number of discrete time intervals, or timesteps. The user specifies the length of each timestep and the number of timesteps in each cycle. The input time dependent coefficients are then characterized by the user for the first cycle, and BFISH repeats the values each succeeding cycle. An obvious time structure would be to use annual cycles with weekly, monthly, or quarterly timesteps depending on the availability of data and desired output resolution. By characterizing the necessary coefficients over a single cycle, calculations may proceed over any number of desired cycles without additional user involvement. The option exists, of course, to set up a single-cycle simulation, with the cycle being arbitrarily long. This would require, however, characterization of the time dependent coefficients over the entire span of the simulation, which could be very tiresome. The cycle concept takes advantage of the cyclical nature of such activities as spawning and migration to economize on input labors without reducing the model's capabilities.

Note that the recruitment rate to a given species at any time t is a function of the total population of that species at some prior time (= the maturation age of that species). That is, no geographical effect on spawning activity is accounted for, as the distribution of the fish at time of spawning ($t = t_s$) does not affect the total number of recruits at time $t = t_s + m_j$. Presently, the new recruits to species j are distributed to the various compartments based upon the fraction of the total population of species j that resides in each compartment at the beginning of the timestep being calculated, i.e.,

$$FR_{ij}(t) = \frac{P_{ij}(t_o)}{\sum_i P_{ij}(t_o)} \quad (t_o \leq t \leq t_1) \tag{2}$$

t_o = time at beginning of present timestep.

t_1 = time at end of present timestep.

No species interactions are modeled, hence the equations for each species may be solved independently. In the following discussion the subscript j is dropped for clarity, with the understanding that the equations apply to and are solved for each species in turn. The intercompartmental migration coefficients are coupling agents between the population equations for each compartment. Thus, equation (1) describes a system of N (N = number of compartments) coupled differential equations for each species:

$$\frac{d\bar{P}}{dt} = A\bar{P} + \bar{K} \tag{3}$$

where

$\bar{P}$ = a column vector of compartment populations, i.e.,

$$\bar{P} = \begin{bmatrix} P_1(t) \\ P_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_N(t) \end{bmatrix}$$

A = (N x N) matrix with the migration coefficients in the off-diagonal elements.

$$A = \left[a_{ik}\right]$$

where

$$a_{ii} = -\left(FC_i(t) + FS_i(t) + XN + EM_i(t)\right)$$

$$a_{ik} = XM_{k \to i}(t)$$

and $\bar{K}$ is a column vector of recruitment rates to each compartment, i.e.,

$$k_i = \text{minimum}\begin{cases} FR_i(t)*RP(t - m)*\sum_{\ell=1}^{N} P_\ell(t - m) \\ FR_i(t)*RCMAX \end{cases}$$

B. Yield Equations

The commercial and sport fishing yields of species j in compartment i during the time interval from t_o to t_1 can be expressed:

$$YIELDC_{ij}(t_o, t_1) = \int_{t_o}^{t_1} EC_i(t)*QC_{ij}(t)*P_{ij}(t)dt \tag{4}$$

$$YIELDS_{ij}(t_o, t_1) = \int_{t_o}^{t_1} ES_i(t)*QS_{ij}(t)*P_{ij}(t)dt \tag{5}$$

where

$YIELDC_{ij}(t_o, t_1)$ = commercial fishing yield of species j in compartment i over the time interval from t_o to t_1.

$YIELDS_{ij}(t_o, t_1)$ = sport fishing yield of species j in compartment i over the time interval from t_o to t_1.

III. NUMERICAL SOLUTION

A. Population Dynamics

The population dynamics problem equation (3) is solved using a fourth order Runge-Kutta numerical integration scheme. If we denote the right-hand side of equation (3) by $f(\bar{P}, t)$, we have for compartment i:

$$\frac{dP_i}{dt} = f\left(P_1(t),\ P_2(t),\ \ldots\ P_N(t),\ t\right) \tag{6}$$

where the species subscript j is dropped for clarity. The computing procedure for the fourth order Runge-Kutta integration of equation (6) involves the use of four auxiliary quantities $XK1_i$, $XK2_i$, $XK3_i$, and $XK4_i$. If we know $P_i(t_o)$, we calculate $P_i(t_1)$ from

$$P_i(t_i) = P_i(t_o) + \frac{1}{6}\left(XK1_i + 2*XK2_i + 2*XK3_i + XK4_i\right) \tag{7}$$

where

$$XK1_i = H*f\left(P_1(t_o),\ P_2(t_o),\ \ldots P_N(t_o),\ t_o\right)$$

$$XK2_i = H*f\left(P_1(t_o) + \frac{XK1_1}{2},\ \ldots\ P_N(t_o) + \frac{XK1_N}{2},\ t_o + \frac{H}{2}\right)$$

$$XK3_i = H*f\left(P_1(t_o) + \frac{XK2_1}{2},\ \ldots\ P_N(t_o) + \frac{XK2_N}{2},\ t_o + \frac{H}{2}\right)$$

$$XK4_i = H*f\left(P_1(t_o) + XK3_1,\ \ldots\ P_N(t_o) + XK3_N,\ t_o + H\right)$$

$$H = t_1 - t_o$$

The stability and accuracy of this method are dependent upon the timestep size H. In general, the timestep size should be sufficiently small so that the higher order terms (order >4) neglected in the solution scheme are negligible. In practice, it is very difficult to know a priori what timestep size to assign a given BFISH analysis. Indeed, at times the timestep size will not be arbitrary, but will have a lower bound imposed by available data. For these reasons, it was considered appropriate to monitor the calculations as they proceed. This is done by calculating and printing out the Collatz coefficient of "sensitiveness":[1]

$$S_i = \frac{XK2_i - XK3_i}{XK1_i - XK2_i} \tag{8}$$

A reasonable timestep size will yield a value of S close to unity. If S is too much larger than unity, a smaller timestep size is desirable. If S is too much smaller than unity, a larger timestep size may allow the user to decrease computational costs while maintaining desired accuracy. Validation runs with the BFISH model yield excellent agreement with exact analytical results for $S \leq 0.5$, and this value is suggested as a rule of thumb. However, S values of twice that size can be tolerated with only minimal loss of accuracy ($\sim$5%).

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[1] Collatz, Lothar. 1966. The numerical treatment of differential equations. Springer-Verlag, New York, Inc., N.Y., 568 p.

B. Yield Equations

The numerical technique used to calculate the fishing yields is presented here in terms of a general yield function. Specific application to commercial and sport fishing yields in the BFISH model merely requires the appropriate effort and catchability coefficient functions.

The general yield function, Y, for a given species and compartment, is defined by:

$$Y(t) = E(t)*Q(t)*P(t) \tag{9}$$

E(t) = fishing effort

Q(t) = fishing catchability coefficient

P(t) = fish population

The fishing yield over some time interval (t_o, t_1) will be the integral of equation (9) over the interval:

$$\text{YIELD} = \int_{t_o}^{t_1} Y(t)dt = \int_{t_o}^{t_1} E(t)*Q(t)*P(t)dt \tag{10}$$

If we assume that Y(t) on the interval can be represented by a third order Taylor series expansion about t_o, we have

$$Y(t) = Y(t_o) + Y'(t_o)\Delta t + Y''(t_o)\frac{\Delta t^2}{2} + Y'''(t_o)\frac{\Delta t^3}{6} \tag{11}$$

for $(t_o \leq t \leq t_1)$

$\Delta t = t - t_o$

which is easily analytically integrated to give:

$$\int_{t_o}^{t_1} Y(t)dt = Y(t_o)H + Y'(t_o)\frac{H^2}{2} + Y''(t_o)\frac{H^3}{6} + Y'''(t_o)\frac{H^4}{24} \tag{12}$$

where $H = t_1 - t_o$.

Thus, if equation (11) is an adequate representation of the yield function $Y(t)$ on (t_o, t_1), and if we can characterize the expansion coefficients, the fishing yield over the interval can be easily calculated from equation (12).

Note that $E(t)$ and $Q(t)$ are input functions of time, and their values and derivatives are known for all time t. At the beginning of a timestep the population $P(t_o)$ is known, and by substitution into equation (3) $P'(t_o)$ is known. Similarly, at the completion of the Runge-Kutta calculations over the timestep $P(t_1)$ and $P'(t_1)$ will be known. Hence, we can easily calculate

$$Y(t_o) = E(t_o)*Q(t_o)*P(t_o) \tag{13}$$

$$\begin{aligned} Y'(t_o) = {} & E(t_o)*Q(t_o)*P'(t_o) + E(t_o)*Q'(t_o)*P(t_o) \\ & + E'(t_o)*Q(t_o)*P(t_o) \end{aligned} \tag{14}$$

$$Y(t_1) = E(t_1)*Q(t_1)*P(t_1) \tag{15}$$

$$\begin{aligned} Y'(t_1) = {} & E(t_1)*Q(t_1)*P'(t_1) + E(t_1)*Q'(t_1)*P(t_1) \\ & + E'(t_1)*Q(t_1)*P(t_1) \end{aligned} \tag{16}$$

By evaluating equation (11) and its first time derivative at the end of the timestep ($t = t_1$) we get

$$Y(t_1) = Y(t_o) + Y'(t_o)H + Y''(t_o)\frac{H^2}{2} + Y'''(t_o)\frac{H^3}{6} \qquad (17)$$

$$Y'(t_1) = Y'(t_o) + Y''(t_o)H + Y'''(t_o)\frac{H^2}{2}$$

Since we know $Y(t_o)$, $Y'(t_o)$, $Y(t_1)$, and $Y'(t_1)$, equations (17) form a system of two equations in two unknowns $\left(Y''(t_o)\right.$ and $\left.Y'''(t_o)\right)$. This system is solved to yield the recursion equations:

$$Y'''(t_o) = -\frac{12}{H^3}\left(Y(t_1) - Y(t_o)\right) + \frac{6}{H^2}\left(Y'(t_1) + Y'(t_o)\right) \qquad (18)$$

$$Y''(t_o) = \frac{2}{H^2}\left(Y(t_1) - Y(t_o)\right) - \frac{H}{3}Y'''(t_o) - \frac{2}{H}Y'(t_o) \qquad (19)$$

We can now complete the expansion equation (11) and calculate the integral equation (12).

This technique is generally applicable to any function on any interval (t_o, t_1) for which the appropriate boundary values and derivatives are known. Specifically, in BFISH, it is used in three different calculations. The first two are the yield function for commercial and sport fishing yields, respectively.

$$\text{YIELDC} = \int_{t_o}^{t_1} EC(t)*QC(t)*P(t)dt \qquad (20)$$

YIELDC = commercial fishing yield

$$\text{YIELDS} = \int_{t_o}^{t_i} ES(t)*QS(t)*P(t)dt \tag{21}$$

YIELDS = sport fishing yield

These yields are calculated for each species and compartment over each timestep. The third application of the technique is in the recruitment calculation, which is discussed in the next section.

The basic assumption implicit in this technique is that the function to be integrated can be adequately described on the interval of interest by a third order Taylor series. Validation runs with BFISH are in excellent agreement with exact analytical results for timestep sizes which yield a Collatz coefficient $S \leq 0.5$. Thus, a high degree of accuracy can be maintained in both the Runge-Kutta and yield calculations by keeping S within the recommended range.

C. Recruitment

As was noted previously, the recruitment rate at time t is dependent upon the spawning population at some previous time (t - m). Specifically, the recruitment rate for a given species at time t is given by

$$\left.\frac{dP(t)}{dt}\right|_{\text{RECRUITMENT}} = \text{minimum}\begin{cases} RP(t - m)*P_{TOT}(t - m) \\ \\ RCMAX \end{cases} \tag{22}$$

RP = reproduction coefficient

P_{TOT} = total population

RCMAX = maximum recruitment rate

Here our recruitment rate as a function of population size is divided into a proportional region and a constant region. If we define a reproduction function

$$RPP(t) = \text{minimum}\begin{cases} RP(t)*P_{TOT}(t) \\ \\ RCMAX \end{cases} \tag{23}$$

we can express the total number of fish recruited to a population over an interval (t_o, t_1) as

$$\Delta P = \int_{t_o}^{t_1} dP = \int_{t_o}^{t_1} RPP(t - m)dt \tag{24}$$

The BFISH model incorporates the concept of a recruitment period for each species. This is some arbitary segment of each time cycle during which the fish are assumed to spawn. The maturation age, m, is input as the number of cycles between the time of spawning and the time when the new fish enter the fishery. Thus, spawning activity during the k^{th} timestep of cycle L will produce new recruits to the population during the k^{th} timestep of cycle L + m. The recruitment period itself is an arbitrary number of timesteps in length, and occurs at the same time each cycle. Numerically, the problem is to characterize the reproduction function, RPP, over the recruitment period for each cycle, and store the characterization for retrieval m cycles later.

Using the same arguments as applied to the yield function, we know RPP and RPP′ at the beginning and end of each timestep, and thus can calculate the integral of the function over the timestep in the same manner using a third order Taylor series. This integral, equation (24), is evaluated for each timestep in the recruitment period. The total number of fish that will be added to the population by the recruitment mechanism m cycles later is

$$\bar{R} = \sum_{i=1}^{N_R} \Delta P = \sum_{i=1}^{N_R} \int_{t_{oi}}^{t_{1i}} RPP(t)dt \tag{25}$$

N_R = number of timesteps in the recruitment period.

t_{oi} = time at beginning of timestep i in the recruitment period.

t_{1i} = time at end of timestep i in the recruitment period.

If we define:

condition 1 $\equiv RP(t)*P_{TOT}(t) < RCMAX$

condition 2 $\equiv RP(t)*P_{TOT}(t) \geq RCMAX$

we can see that at the end of each recruiment period, BFISH will have calculated:

$$RPP(t_I) = \begin{cases} RP(t_I)*P_{TOT}(t_I) & ; \text{ condition 1} \\ RCMAX & ; \text{ condition 2} \end{cases} \tag{26}$$

$$RPP'(t_I) = \begin{cases} RP(t_I)*P'_{TOT}(t_I) + RP'(t_I)*P_{TOT}(t_I) & ; \text{ condition 1} \\ \\ 0 ; \text{ condition 2} & \end{cases} \tag{27}$$

$$RPP(t_F) = \begin{cases} RP(t_F)*P_{TOT}(t_F) & ; \text{ condition 1} \\ \\ RCMAX & ; \text{ condition 2} \end{cases} \tag{28}$$

$$RPP'(t_F) = \begin{cases} RP(t_F)*P'_{TOT}(t_F) + RP'(t_F)*P_{TOT}(t_F) & ; \text{ condition 1} \\ \\ 0 ; \text{ condition 2} & \end{cases} \tag{29}$$

$$\bar{R} = \int_{t_I}^{t_F} RPP(t)dt = \sum_{i=1}^{N_R} \int_{t_{oi}}^{t_{1i}} RPP(t)dt \tag{30}$$

where t_I = time at the beginning of the recruitment period.

t_F = time at the end of the recruitment period.

Next, we assume that the function RPP(t) can be represented over the entire recruiment period (t_I, t_F) by a fourth order Taylor series expansion about t_I:

$$RPP(t) = RPP(t_I) + RPP'(t_I)\Delta t + RPP''(t_I)\frac{\Delta t^2}{2} + RPP'''(t_I)\frac{\Delta t^3}{6}$$
$$+ RPP''''(t_I)\frac{\Delta t^4}{24} \quad (t_I \le t \le t_F) \tag{31}$$

$$\Delta t = t - t_I$$

Differentiating equation (31) with respect to time yields

$$RPP'(t) = RPP'(t_I) + RPP''(t_I)\Delta t + RPP'''(t_I)\frac{\Delta t^2}{2} + RPP''''(t_I)\frac{\Delta t^3}{6} \tag{32}$$

and integrating equation (31) over the recruitment period yields:

$$\bar{R} = RPP(t_I)\Delta t_R + RPP'(t_I)\frac{\Delta t_R{}^2}{2} + RPP''(t_I)\frac{\Delta t_R{}^3}{6}$$
$$+ RPP'''(t_I)\frac{\Delta t_R{}^4}{24} + RPP''''(t_I)\frac{\Delta t_R{}^5}{120} \tag{33}$$

where $\Delta t_R = t_F - t_I$

Note that we know $RPP(t_I)$, $RPP'(t_I)$, $RPP(t_F)$, $RPP'(t_F)$, and $\bar{R}$. Thus, by evaluating equations (31) and (32) at $t = t_F$, and augmenting the two resulting equations with equation (33), we have a system of 3 equations in 3 unknowns $\left(RPP''(t_I),\ RPP'''(t_I),\text{ and } RPP''''(t_I)\right)$. This system can be solved to yield the recursion equations:

$$RPP''''(t_I) = \frac{720}{\Delta t_R{}^5}\bar{R} - \frac{360}{\Delta t_R{}^4}\left(RPP(t_F) + RPP(t_I)\right)$$
$$+ \frac{60}{\Delta t_R{}^3}\left(RPP'(t_F) - RPP'(t_I)\right) \tag{34}$$

$$
\begin{aligned}
RPP'''(t_I) &= \frac{6}{\Delta t_R^{\,2}} \left(RPP'(t_F) + RPP'(t_I)\right) \\
&= - \frac{12}{\Delta t_R^{\,3}} \left(RPP(t_F) - RPP(t_I)\right) - \frac{\Delta t_R}{2} RPP''''(t_I)
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
RPP''(t_I) &= \frac{2}{\Delta t_R^{\,2}} \left(RPP(t_F) - RPP(t_I)\right) - \frac{2}{\Delta t_R} RPP'(t_I) \\
&\quad - \frac{\Delta t_R}{3} RPP'''(t_I) - \frac{\Delta t_R^{\,2}}{12} RPP''''(t_I)
\end{aligned}
\tag{36}
$$

This completes the characterization of the Taylor series expansion of the reproduction function RPP(t) over the recruitment period. The coefficients of the expansion are stored, and m cycles in the future, recalled. The recruitment rate at time t for a given cycle is then:

$$RC(t) = RPP(t - m) \qquad t_I + m \leq t \leq t_F + m \tag{37}$$

$$
\begin{aligned}
RC(t) = 0 \qquad & t < t_I + m \\
& t > t_F + m
\end{aligned}
\tag{38}
$$

The total recruitment rate for a given species is then divided among the compartments based upon the fraction of the total species population that resides in each at the beginning of the timestep.

Thus, after the first m cycles the recruitment mechanism will be fully self-contained. Some provision must be made, however, to recruit fish to the population from time = 0. To accomplish this, the user specifies a constant recruitment rate, RCINIT, that is used during the recruitment periods of cycles 1 through m. After the m^{th} cycle, this initial recruitment rate is superseded by the BFISH calculated recruitment rate.

D. Internal Management Schemes

The BFISH model has the capability of keeping separate population and yield totals for a user-specified management area, which is a subset of the total number of compartments used in the model. Commercial fishing effort for each compartment may be supplied by the user or calculated by BFISH. Within the management area, additional constraints may be placed upon the commercial fishing effort in the form of effort or yield ceilings. Some inter-species numerical coupling may exist as a result of the commercial fishing fleet keying on more than one species, or multi-species yield ceilings being specified by the user. As described below, when this coupling exists it is manifested during step changes in the applied effort function across timestep boundaries, and does not affect the solution procedure described earlier.

The options available to the user for the distribution of commercial fishing effort and the management of the managed area are described below:

1) MANAGEMENT OPTION 1

When the first management option is specified, the commercial fishing effort function must be supplied by the user for each compartment separately. A management area may be specified (some arbitrary subset of the total compartment set), for which population and yield subtotals will be kept, but no special calculations or constraints will be applied.

2) MANAGEMENT OPTION 2

When the second management option is specified, BFISH will internally calculate the commercial fishing effort in each compartment subject to an effort ceiling placed on each managed compartment. The user must supply the total commercial fishing effort function (total effort = effort summed over all compartments) and the effort ceiling function for each managed compartment.

The total commercial fishing effort function is utilized as a step function with the total effort assumed constant over each timestep. Hence, if we define

TE(t) = total commercial fishing effort function supplied by the user

BFISH will calculate the total fishing effort to be divided up among the various compartments from:

$$
\begin{array}{ll}
\text{TEFORT} = \text{TE}(t_1) & t_1 < t \leq t_2 \\
\text{TEFORT} = \text{TE}(t_2) & t_2 < t \leq t_3 \\
\vdots & \vdots \\
\text{TEFORT} = \text{TE}(t_k) & t_k < t \leq t_{k+1}
\end{array}
\tag{39}
$$

where

TEFORT = total commercial fishing effort used by BFISH

t_k = time at beginning of k^{th} timestep.

Thus, the total commercial fishing effort function supplied by the user differs from the other time dependent input functions in that it is never evaluated between timestep boundaries.

It is assumed that the commercial fishing fleet will key on the movements of their target species, and thus the most commercial effort will be found where there are the most fish. It is recognized, however, that the fleet may not instantaneously recognize fish movement, and a lag time may exist between the real fish distribution and the response of the commercial fleet. Consequently, a lag time option is included in the BFISH model, invoked by assigning a value of 0 or 1 to the interger variable ILAG. If ILAG = 0, the commercial fishing effort in each compartment for each timestep is calculated based on the total effort available for that timestep and the fraction of the total target population that resides in each compartment at the beginning of the timestep. If ILAG = 1, the commercial effort is distributed based on the target population distribution at the beginning of the previous timestep, i.e.,

$$\text{If ILAG} = 0; \; E_i(t) = \frac{\text{PTARGT}_i(t_k)}{\text{TTARGT}(t_k)} * \text{TE}(t_k) \qquad t_k < t \leq t_{k+1}$$

$$\text{If ILAG} = 1; \; E_i(t) = \frac{\text{PTARGT}_i(t_{k-1})}{\text{TTARGT}(t_{k-1})} * \text{TE}(t_k) \qquad t_k < t \leq t_{k+1} \tag{40}$$

$E_i(t)$ = commercial fishing effort applied to compartment i at time t

t_k = time at beginning of timestep k

$\text{PTARGT}_i(t)$ = target population in compartment i at time t

$\text{TTARGT}(t) = \sum_i \text{PTARGT}_i(t)$

Note that $E_i(t)$ is constant over each timestep with step changes at the timestep boundaries. The target population may be composed of one or several constituent species. Thus, the movement of the commercial fishing fleet may be modeled as a function of the movement of the target species of fish. If this is all that is required, the user would specify a null set of managed compartments. If, however, a management area is used, then the compartments within the managed area are subject to the additional constraint of user-supplied effort ceilings.

$$CEILNG_i(t) = CL_i(t_k) \quad t_k < t \leq t_{k+1} \tag{41}$$

$CL_i(t)$ = ceiling function for compartment i evaluated at time t
$CEILNG_i(t)$ = effort ceiling used by BFISH for compartment i at time t

Note that the effort ceiling function, like the total effort function, is used as a step function constant across each timestep. The commercial fishing effort calculation will then proceed as follows:

a) Calculate the commercial fishing effort in each compartment from equation (40).

b) Check each managed compartment to see if the calculated fishing effort exceeds the effort ceiling for that compartment.

c) If the calculated effort exceeds the ceiling, set the effort equal to the ceiling and save the amount of excess effort that must be distributed elsewhere. Remove the compartment from further calculations.

d) After (b) and (c) have been completed for all compartments, distribute the accumulated excess effort among the compartments still in the calculation based on their respective target populations.

e) Go back to (b).

This iterative process continues until there is no more excess effort at the end of step (c).

3) MANAGEMENT OPTION 3

When the third management option is specified, BFISH will again internally calculate the distribution of the total commercial fishing effort from equation (40), except that a yield ceiling is placed on each compartment in the management area. This yield ceiling is not time dependent, and is used as a cycle-to-date yield limit. Thus, compartment i is removed from the calculations if

$$YMANAG_i(t_k) \geq YCEILNG_i$$

$YMANAG_i(t)$ = total commercial fishing yield of managed species taken during the present time cycle in compartment i up to time t.

t_k = time at the beginning of the k^{th} timestep.

$YCEILNG_i$ = cycle-to-date yield ceiling for compartment i

The managed species may be all the species modeled or some subset thereof. Note that the cycle-to-date yield is checked and the effort is distributed at the beginning of each timestep. It is therefore possible for the commercial fishing yield in a managed compartment to be less than the applied ceiling at the beginning of the timestep, and exceed the ceiling during the timestep. This will be flagged at the beginning of the next timestep, and no more commercial fishing effort will be applied to that compartment for the remainder of the time cycle. Naturally, all cycle-to-date totals are reset to zero at the beginning of each time cycle.

For the third management option, the user must supply the total commercial fishing effort function, the constituent compartments in the management area, the yield ceiling for each managed compartment, the constituent species in the target fish population, and the constituent species in the managed fish population, and the ILAG value.

IV. INPUTS

A. General Information

The majority of the inputs required to run a BFISH analysis are characterizations of the time dependent coefficients. As mentioned earlier, these functions need only be characterized over the first time cycle, and are repeated for each subsequent cycle. Thus, one must completely describe each time dependent input function over the interval from (time = zero) to (time = end-of-cycle). Using f as a general function for demonstration purposes, we must describe

$$f(t) \qquad 0 \leq t \leq t_F$$

where t_F = time at end-of-cycle

There are two alternative ways to describe f(t) over the cycle available to the BFISH user. First, if f(t) is smooth and well-behaved, it might be possible to represent f as a polynomial in time:

$$f(t) = a_1 + a_2 t + a_3 t^2 + \dots a_{n+1} t^n \tag{42}$$

$$= \sum_{i=o}^{n} a_{i+1} \; t^i$$

$$(0 \leq t \leq t_F)$$

In this case the user need only specify n, the order of the polynomial, and the (n + 1) coefficients a_i.

The second type of representation is to specify explicitly what value f(t) is to take at the timestep boundaries. BFISH will then treat f(t) as a piecewise linear function with as many "pieces" as there are timesteps. The value of f(t) at any time during the timestep will fall along the line connecting the user supplied values at the timestep boundaries. For example, f(t) would be evaluated on (t_k, t_{k+1}) from

$$f(t) = f(t_k) + \left[\frac{t - t_k}{t_{k+1} - t_k}\right] * \left[f(t_{k+1} - f(t_k)\right] (t_k \leq t \leq t_{k+1}) \qquad (43)$$

where

$f(t_k)$ = user supplied value of f(t) at the beginning of the k^{th} timestep

t_k = time at the beginning of the k^{th} timestep.

The end-of-timestep value for any timestep is the same as the beginning-of-timestep value for the next timestep. Thus, by supplying the beginning of timestep values for each timestep in the cycle, a continuous piecewise linear function is completely described across the cycle.

It is also realized that many parameter estimation procedures are not well suited for calculating time-dependent variables, and estimating some of the coefficients may only be tractable if one assumes they remain constant over a timestep. Thus, the user has the option of making the migration coefficients, commercial fishing catchability coefficients, or sport fishing catchability coefficients piecewise constant. Using this option, the function is represented in either of the two manners described above, but is evaluated at the beginning of each timestep and held constant over the timestep. This option is invoked by assigning a value of 2 to the integer variables IMIG, IQC, and IQS for the migration coefficients, commercial fishing catchability coefficients, and sport fishing catchability coefficients, respectively. By assigning a value of 1 to the above integer variables, the user has elected to represent the corresponding coefficient as a continuous function.

When describing an input function, the input card is, in general, utilized as follows:

COLS 1 - 3 : species number

COLS 4 - 6 : compartment number

COLS 7 - 9 : type of representation

(1 = polynomial)

(2 = explicit)

COLS 10 - 12 : order of the polynomial (if type 1) or card number (if type 2)

COLS 13 - 23, 24 - 34, 35 - 45, 46 - 56, 57 - 67, 68 - 78 : fixed decimal input values for polynomial coefficients (if type 1) or modal values (if type = 2).

Thus, columns 1-12 hold four integer variables of field width 3, and columns 13-78 hold six fixed decimal variables of field width 11. This breakdown is reflected in the standard BFISH coding form shown in Figure 1.

As an example of a polynomial input, assume we wish to represent f(t) as a fourth order polynomial:

$$f(t) = 5. + 0.06\ t - 0.7\ t^3 + 0.0006\ t^4 \qquad (44)$$

$$(0 \leq t \leq t_F)$$

where f(t) describes some time dependent coefficient for species 1 and compartment 3. In general, the species will be specified once and need not be repeated for all inputs pertaining to that species. Assume this has been done on some previous card. The necessary input card representing f(t) would then appear as in Figure 2A. Note that the compartment number, type of representation, order of the polynomial, and the polynomial coefficients all appear on the same input card. This format is characteristic of polynomial (type 1) variables. Since there is space on each card for 6 fixed decimal coefficients, the user is limited to polynomials of order 5 or less. The effort ceiling polynomial, as noted below, is restricted to order 4 or less.

Figure 1.1 - BFISH coding form.

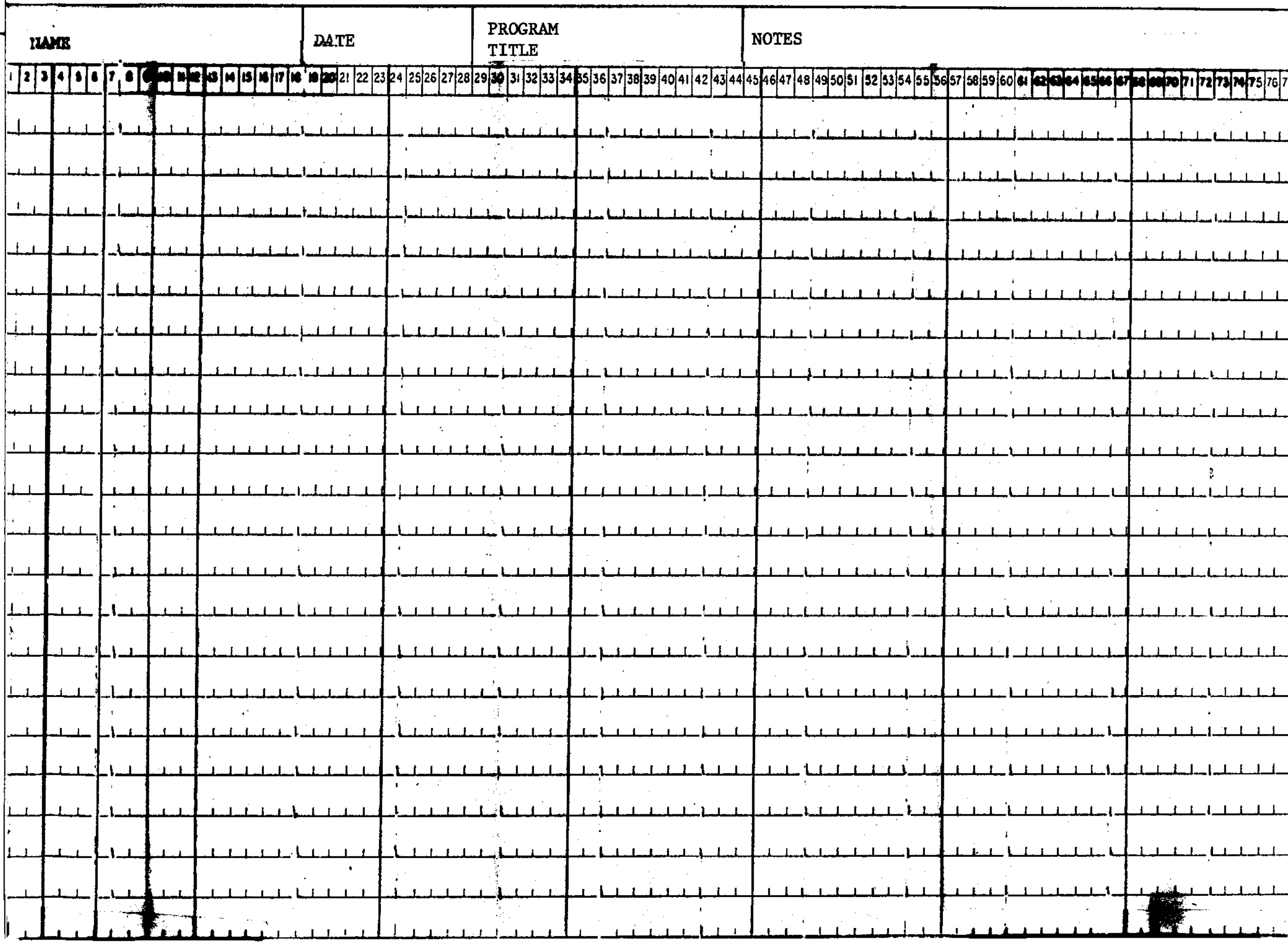

Figure 2A.--Type 1 representation.

3	1	4	5.	.06	0.	-.7	.0006

fig. 2A

Figure 2B.--Type 2 representation.

3	2						
		1	0.	30.	30.	60.	
		2	60.	90.	30.	30.	
		3	30.	15.			

Now suppose the user has decided to break up each cycle into 10 timesteps, and wishes to represent f(t) over each cycle as shown in Figure 3. In this case f would be explicitly represented (type 2) and the necessary input cards would be as shown in Figure 2B. Note that the set of cards with the input nodal values follows the card describing the compartment number and type of representation, each card containing nodal points is numbered, and a maximum of four nodal values appears on each card. These characteristics will always be true for explicit (type 2) variables. The number of timesteps per cycle, and hence the number of cards necessary to list all the nodal values, is arbitrary, but a nodal value must appear for each timestep.

If f(t) were a function describing a migration coefficient, commercial fishing catchability coefficient, or sport fishing catchability coefficient, the user could invoke the piecewise constant option by setting IMIG, IQC, or IQS = 2. In this case, f(t) would be evaluated at the beginning of each timestep and held constant at that value throughout the timestep, i.e.,

$$f(t) = f(t_k) \quad t_k < t \leq t_{k+1}$$

t_k = time at beginning of timestep k

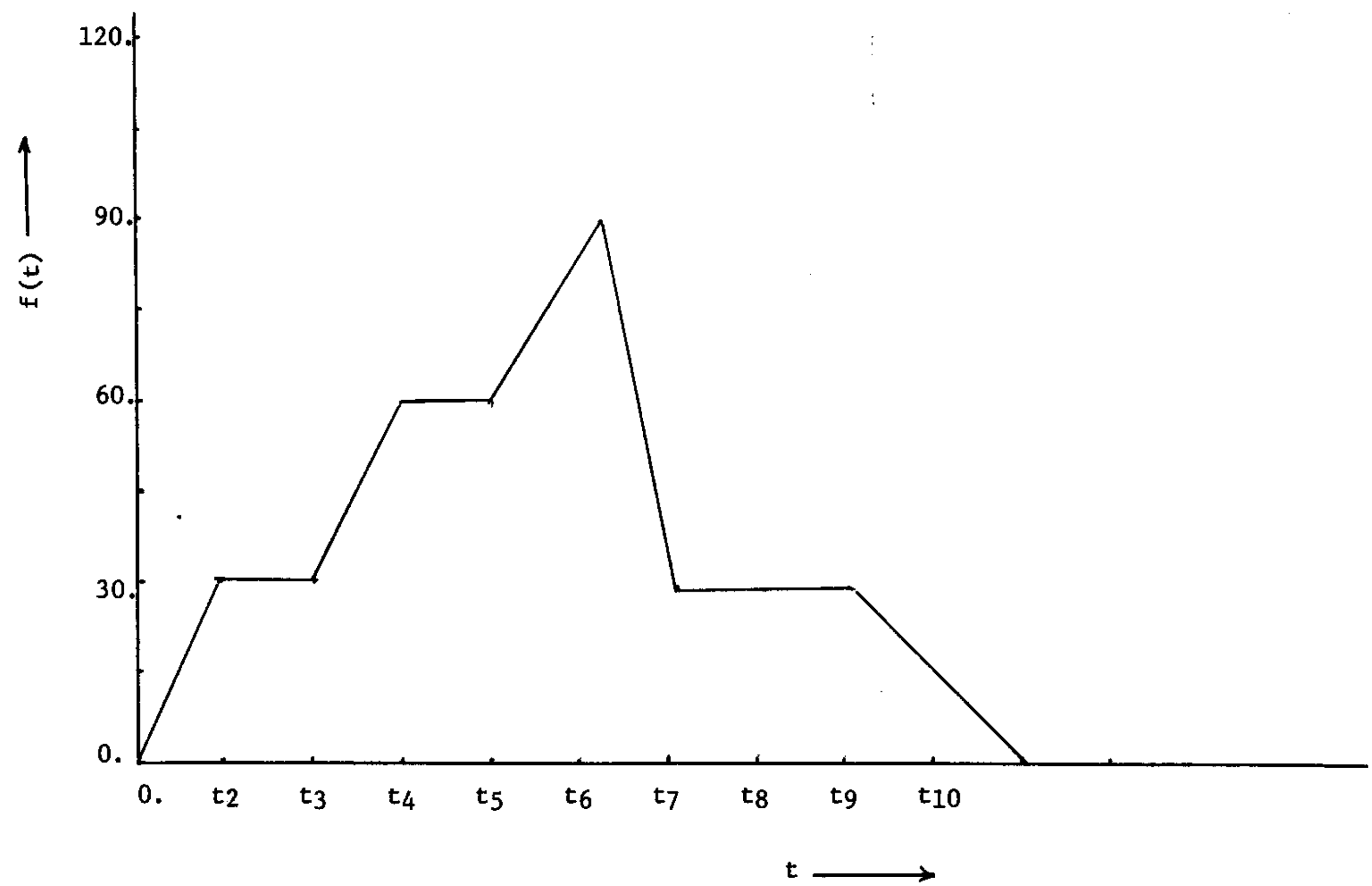

Figure 3.--Time dependence of f(t).

B. Input Deck Format

The format for the input deck necessary to run a BFISH analysis is shown in Figure 4, which is discussed in the following paragraphs.

1. Model structure

The first two cards of any BFISH input deck set up the model structure. Referring to Figure 4, we have:

NSP = number of species (≤ 3)

NCM = number of compartments (≤ 50)

NCY = number of cycles to be modeled

NST = Number of timesteps per cycle

H = length of each timestep

NMC = number of managed compartments ($\leq$ NCM)

(= number of compartments in the management area)

MOPT = management option

(= 1, 2, or 3)

ILAG = lag time (in timesteps) between the target fish distribution and the commercial fishing fleet distribution (= 0 or 1), used only if MOPT = 2 or 3

$$\text{IMIG} = \begin{cases} 1 \rightarrow \text{continuous migration coefficient functions} \\ 2 \rightarrow \text{piecewise constant migration coefficient functions} \end{cases}$$

Figure 4.--BFISH input format.

MODEL STRUCTURE INPUTS:

1-3	4-6	7-9	10-12	13-23
NSP	NCM	NCY	NST	H
NMC	MOPT	ILAG		
IMSG	IQC	IQS		
IKEY	IKS_1	IKS_2	IKS_3	
[illegible]	[illegible]	[illegible]	[illegible]	

IF MOPT = 1

FOR EACH COMPARTMENT (i = 1 to NCM): COMMERCIAL FISHING EFFORT INPUTS

1-3	4-6	7-9	10-12	13-23
	C_i	T_i	0[1]	coefficients[1] →
			cd[2]	nodal values[2] →

IF MOPT = 2:

TOTAL COMMERCIAL FISHING EFFORT INPUTS:

1-3	4-6	7-9	10-12	13-23
		T	0[1]	coefficients[1] →
			cd[2]	nodal values[2] →

FOR EACH MANAGED COMPARTMENT (k = 1 to NMC): EFFORT CEILING INPUTS

1-3	4-6	7-9	10-12	13-23
	C_k	T_k	0[1]	coefficients[1] →
			cd[2]	nodal values[2] →

IF MOPT = 3:

TOTAL COMMERCIAL FISHING EFFORT INPUTS:

Columns 1–6	7–9	10–12	13–23	24–34	35–45	46–51
	T	0[1]	coefficients[1] →			
		cd[2]	nodal values[2] →			

FOR EACH MANAGED COMPARTMENT (k = 1 to NMC): YIELD CEILING

Columns 1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–51
	C_k			Y_k			

FOR EACH COMPARTMENT (i = 1 to NCM): SPORT FISHING EFFORT INPUTS

Columns 1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–51
	C_i	T_i	0[1]	coefficients[1] →			
			cd[2]	nodal values[2] →			

Figure 4.--Continued.

FOR EACH SPECIES (j = 1 to NSP):
RECRUITMENT AND NATURAL MORTALITY INPUTS:

1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–
S_j	[redacted]	[redacted]	m_j	$RCINIT_j$	$RCMAX_j$	$XNMORT_j$	

REPRODUCTION COEFFICIENT INPUTS

1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–
		T_j	0[1]	coefficients[1] →			
			cd[2]	nodal values[2] →			

FOR EACH COMPARTMENT (i = 1 to NCM):
COMMERCIAL FISHING CATCHABILITY COEFFICIENT INPUTS:

1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–
	C_i	T_{ij}	0[1]	coefficients[1] →			
			cd[2]	nodal values[2] →			

FOR EACH COMPARTMENT (i = 1 to NCM):
SPORT FISHING CATCHABILITY COEFFICIENT INPUTS

1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–
	C_i	T_{ij}	0[1]	coefficients[1] →			
			cd[2]	nodal values[2] →			

Figure 4.--Continued.

FOR EACH COMPARTMENT (I = 1 to NCM):

MIGRATION INPUTS

1–3	4–6	7–9	10–12	13–23	24–34	35–45
	C_i	T_{ij}	MTO_{ij}	POP_{ij}		

FOR EACH COMPARTMENT MIGRATED TO (k = 1 to MTO_{ij})

MIGRATION COEFFICIENT INPUTS:

1–3	4–6	7–9	10–12	13–23	24–34	35–40
		CTO_{ijk}	0[1] / cd[2]	coefficients[1], or nodal values[2] →		

1–3	4–6	7–9	10–12	13–23	24–34	35–45	46–51
*							

$$IQC = \begin{cases} 1 \rightarrow \text{continuous commercial fishing catchability coefficient functions} \\ 2 \rightarrow \text{piecewise constant commercial fishing catchability coefficient functions} \end{cases}$$

$$IQS = \begin{cases} 1 \rightarrow \text{continuous sport fishing catchability coefficient functions} \\ 2 \rightarrow \text{piecewise constant sport fishing catchability coefficient functions} \end{cases}$$

IKEY = number of species in target fish population (≤ 3), used only if MOPT = 2 or 3

IKS_i = species number of the i^{th} target species (i = 1 to IKEY)

IYKEY = number of species in the managed population, i.e., those species that will count toward the commercial yield limit (≤ 3), used only if MOPT = 3

IYS_i = species number of the i^{th} managed species (i = 1 to IYKEY)

Only the commercial fishing yield is counted toward the yield limit. Thus, if we want the commercial fishing fleet to key on a target population consisting of the summed population of three different species, but our management scheme calls for an upper limit on the number of fish that can be taken commercially from species number 2 only within our management area, we would set MOPT = 3, NSP = 3, IKEY = 3, IKS_1 = 1, IKS_2 = 2, IKS_3 = 3, IYKEY = 1, IYS_1 = 2.

2. Commercial fishing effort and ceiling inputs

This section of the input deck follows the model structure cards, and will differ in format for different management options. The three alternative (and mutually exclusive) formats are:

a) If MOPT = 1, the commercial fishing effort must be supplied for each compartment separately. Referring to Figure 4, we define:

   C_i = compartment number

   T_i = type of representation (1 or 2, for polynomial or explicit representation, respectively)

   0 = order of the polynomial

   cd = card number

   The superscript 1 denotes variables that are only required if T = 1, and the superscript 2 denotes variables that are only required if T = 2. The first NMC compartments listed will constitute the management area. No special calculation will take place for the managed compartments, but separate population and yield subtotals will be kept for the management area. Compartment numbers must range from 1 to NCM, but need not be listed in order. If T = 2 for any compartment, NST nodal values must be listed, four to a card. Obviously, if NST > 4, there will be more than one card involved in supplying all NST nodal values. Each of these cards is numbered.

b) If MOPT = 2, the user must supply the total commercial fishing effort function and, for each managed compartment, the effort ceiling function. Effort ceiling inputs must be supplied for NMC compartments. Any compartment number not specified in the effort ceiling input set is assumed to be out of the management area (and hence has no effort ceiling). Thus, if NCM = 10 and NMC = 3, three effort ceiling functions must be supplied. Suppose the three C_k supplied are 4, 7, and 9. Then compartments 4, 7, and 9 constitute the management area. Compartments 1, 2, 3, 5, 6, 8, and 10 are unmanaged. If T = 1, 0 for the effort ceiling function must be ≤ 4.

c) If MOPT = 3, the total commercial fishing effort function is again required. In addition, the user must supply the yield ceiling placed on each managed compartment. This is a single quantity, not a function, and is specified by Y_k for compartment C_k in Figure 4. **If the total commercial fishing yield (of the species specified as managed species) in compartment C_k ever exceeds Y_k during any** one time cycle, commercial fishing effort in C_k is set to zero for the remainder of the cycle.

3. Sport fishing effort inputs

For each compartment, the sport fishing effort function is specified.

4. Species dependent inputs

The following input requirements describe the species dependent inputs for a single species. The species number for each set is specified as the first card of the set. NSP such sets must be supplied, one for each species.

a) Recruitment and natural mortality

Referring to Figure 4, the following definitions are made:

S_j = species number

$R1_j$ = first timestep in the recruitment period for species S_j

$R2_j$ = last timestep in the recruitment period for species S_j

m_j = maturation age (in number of cycles) for species S_j

$RCINIT_j$ = initial recruitment rate (constant recruitment rate for the first m_j cycles) for species S_j

$RCMAX_j$ = maximum recruitment rate for species S_j

$XNMORT_j$ = natural mortality coefficient for species S_j

b) Reproduction coefficient

The reproduction coefficient function is supplies for species S_j. Note that this function is only evaluated during the recruitment period for species S_j, and hence must only be valid on the interval (t_{F1_j}, t_{R2_j+1}).

S_j = species number

$R1_j$ = first timestep in the recruitment period for species S_j

$R2_j$ = last timestep in the recruitment period for species S_j

m_j = maturation age (in number of cycles) for species S_j

XN_j = natural mortality coefficient for species S_j.

b) Reproduction coefficient

The reproduction coefficient function is supplies for species S_j. Note that this function is only evaluated during the recruitment period for species S_j, and hence must only be valid on the interval (t_{R1_j}, t_{R2_j+1}).

c) Commercial fishing catchability coefficient

This variable is both species and compartment dependent. The commercial catchability coefficient function is supplied for each compartment C_i for each species S_j.

d) Sport fishing catchability coefficient

This function is supplied for each compartment C_i for each species S_j.

e) Migration coefficients

The format for these inputs differs slightly from the other functional inputs. For each species S_j a migration set is described for each compartment C_i. Referring to Figure 4, the following definitions apply:

T_{ij} = type of function representation for species S_j and compartment C_i (= 1 or 2).

MTO_{ij} = the number of different compartments that species S_j migrates to from compartment C_i during the course of a time cycle (≤ 8).

POP_{ij} = initial population of species S_j in compartment C_i.

Then, for k = 1, MTO_{ij}, the migration coefficient functions are supplied, where:

CTO_{ijk} = the compartment number of the k^{th} compartment migrated to by species S_j from compartment C_i.

If T_{ij} = 1, CTO_{ijk} is followed by the order of the polynomial and the polynomial coefficients to describe the time dependence of the migration coefficient for species S_j migrating from compartment C_i to compartment CTO_{ijk}. If T_{ij} = 2, card number one of the nodal set follows on the same card as CTO_{ijk}. If additional cards are required (NST > 4), CTO_{ijk} is repeated on each card in addition to supplying the card number and the next 4 nodal values.

5. End of file

A star ('*') is placed in column one of an otherwise blank card to denote the end of the input data for the BFISH program. This card is placed immediately after the last card in the input deck, but before any additional record or file marks required by the operating system.

C. Rules for coding

BFISH is written in FORTRAN, and there are some characteristics common to most FORTRAN compilers that are relevant to the input file format. Specifically:

1. Right adjust all integer variables. FORTRAN will add zeroes to fill up the field. For example, if the species number, $S_j = 3$, is punched in column 2 instead of column 3 of the appropriate input card, BFISH will read it as $S_j = 30$. Thus, all values appearing in the four integer fields (columns 1-3, 4-6, 7-9, and 10-12) should be right adjusted.

2. Blanks are interpreted as zeroes. Thus, leaving any field blank is equivalent to giving that variable a value of zero.

3. Decimals must be supplied to fixed decimal inputs. It is possible, but not probable, that the value will be interpreted correctly if the decimal is omitted. The issue can be avoided by supplying decimals in the proper places in all values punched in fixed decimal fields (columns 13-23, 24-34, 35-45, 46-56, 57-67, and 68-78).

V. OUTPUT

A. Echo Check

The first section of the BFISH output file is a complete list of all the input parameters with each value associated with its function in the program. These values should be carefully checked to see if the user's intentions have been accurately transmitted to the BFISH model. It is expected that most of the problems that arise in using the model will be at the input level, and the echo section of the output should be scrutinized for accuracy each time the model is employed.

BFISH reads in the nodal points of explicitly represented variables, in groups of 4, as they are needed during execution. Thus, not all points are kept in storage (if NST > 4) at any one time. These nodal points are printed out as they are read. The first four points appear in the echo section. Any additional points are identified and printed out as they are read during execution. Thus, the timestep output may be intermittently interrupted to print out sets of nodal points as they are needed.

B. Timestep Output

The timestep output is printed out at the end of each timestep. This is an important consideration, as BFISH contends with discontinuous functions (e.g., commercial fishing effort when MOPT = 2 or 3) and discontinuous first derivatives (e.g., any explicitly represented variable) across timestep boundaries. The values printed at time t are consistent with an incremental approach to t from the negative side (i.e., are equivalent to the appropriate values at time $(t - \varepsilon)$ where ε is arbitrarily small).

First the time is identified by the following four variables:

ICYCLE = time cycle number

ISTEP = timestep number within this cycle

TIME = time expired within this cycle

TTOTAL = total time expired

The fishing population and yield outputs are then presented for each species in turn. Within each species set, the fishing efforts, catchability coefficients, recruitment rates, populations, and yields are printed for each compartment. In addition, the Collatz coefficient, S, is printed for each compartment for the calculation just completed. This coefficient is defined by equation (8), and should be used as a measure of the appropriateness of the timestep size. If the population gradient is zero, equation (8) will be undefined, and S is set equal to -1.

Each species set continues with a printout of the population and yields summed over all compartments and summed over the management area. Cycle-to-date yields are also calculated and printed. These outputs are repeated for each species at the end of each timestep.

VI. SYNOPSIS OF ROUTINES

BFISH was constructed in modular form to facilitate program updates. This type of construction yields many function and subprogram subroutines, each isolating a separate calculational step. The relationships between the constituent routines are shown in Table 1. These routines are listed and briefly described below:

A. Main Driver

The main driver serves as the focal point for the BFISH modules. The main procedure steps through time, performing the Runge-Kutta and yield integration over each timestep. An outline of the main driver program flow appears in Figure 5.

B. Function EVAL

Function EVAL evaluates a specified function at a specified time. This function is called to calculate the values of the time dependent coefficients at specific times during the time cycle. An outline of the EVAL program flow appears in Figure 6.

C. Function PRIME

Function PRIME evaluates the first time derivative of a specified function at a specified time. This function is called to calculate the first derivatives of time dependent coefficients necessary to carry out the yield function integrations. An outline of the PRIME program flow appears in Figure 7.

Table 1.--Routine hierarchy in BFISH.

Function or program	Calls	Is called by
MAIN	DATIN READ4 SUMGRT EFFORT EVAL XGRATE RPOP RCRUIT PRIME POPDOT RSOLVE	
EVAL	POLY	MAIN EFFORT XGRATE
PRIME	POLY	MAIN
POLY		EVAL PRIME
XGRATE	EVAL RPOP	MAIN
RSOLVE		MAIN
SUMGRT		MAIN
RPOP		MAIN XGRATE
POPDOT		MAIN
RCRUIT		MAIN
EFFORT	EVAL	MAIN
DATIN	SKIP	MAIN
READ4	FIND	MAIN
FIND	SKIP	READ4
SKIP		FIND

Figure 5.--Main program outline.

I. Initialize variables.

II. For each timestep:

  A. For each species:

    1. For each compartment:

      a. Calculate XK1

      b. Store the beginning-of-timestep commercial and sport fishing yield function values for later integration.

    2. Store the beginning-of-timestep reproduction function value for later integration.

    3. For each compartment, calculate XK2.

    4. For each compartment, calculate XK3.

    5. For each compartment:

      a. Calculate XK4.

      b. Calculate end-of-timestep opulation.

    6. For each compartment:

      a. Calculate end-of-timestep commercial and sport fishing yield function values.

      b. Integrate the commercial and sport fishing yield functions over the timestep.

    7. Calculate the end-of-timestep reproduction function value and integrate the reproduction function over the timestep.

    8. If this is the last timestep in the recruitment period, push the RSTACK array and calculate the recruitment function coefficients for next cycle.

  B. Increment counters and zero out step totals.

Figure 6.—Function EVAL outline.

I. Branch to the appropriate function.

II. Branch to the appropriate type of representation.

   A. If TYPE = 1:

      1. Evaluate the polynomial at time t.

      2. Return.

   B. If TYPE = 2:

      1. Evaluate the function as if it were linear across the timestep.

      2. Return.

Figure 7.--Function PRIME outline.

I. Branch to the appropriate function.

II. Branch to the appropriate type of representation.

   A. If TYPE = 1:

      1. Evaluate the first derivative of the polynimial at time t.

      2. Return.

   B. If TYPE = 2.

      1. Evaluate the first derivative of the function as if the function was linear (first derivative constant) across the timestep.

      2. Return.

D. Function POLY

Function POLY evaluates an arbitrary polynomial in time given the polynomial coefficients and the time.

E. Function XGRATE

Given some arbitrary function values $f(t_1)$, $f'(t_1)$, $f(t_2)$, and $f'(t_2)$, XGRATE uses the Taylor series method described earlier to calculate and return the value of the integral:

$$\text{XGRATE} = \int_{t_1}^{t_2} f(t)dt$$

F. Subroutine RSOLVE

Subroutine RSOLVE takes the stored reproduction data for a given species and time cycle and calculates the coefficients of a polynomial in time that describes the recruitment rate to the species due to spawning activity during the stored cycle.

G. Subroutine SUMGRT

Subroutine SUMGRT calculates (see equation 1), for some specific species and time t:

$$\text{EMIGRT}_i(t) = \sum_{i \neq k} \text{XM}_{i \to k}(t)$$

$$\text{SMIGRT}_i(t) = \sum_{i \neq k} \text{XM}_{k \to i}(t) * P_k(t)$$

H. Function RPOP

Function RPOP returns the augmented population values necessary to calculate the Runge-Kutta coefficients.

I. Function POPDOT

Function POPDOT evaluates equation (1) given the necessary coefficients.

J. Function RCRUIT

For a given species, RCRUIT divides up the total recruitment rate among the various compartments.

K. Subroutine EFFORT

Subroutine EFFORT calculates and distributes the commercial fishing effort among the various compartments based upon the user selected management option. An outline of the EFFORT program flow appears in Figure 8.

L. Subroutine DATIN

Subroutine DATIN makes the initial read from the input file. An outline of the DATIN program flow appears in Figure 9.

M. Subroutine READ4

Subroutine READ4 is called to read the next four nodal points of the explicitly represented variables when they are required. An outline of the READ4 program flow appears in Figure 10.

Figure 8.--Subroutine EFFORT outline.

I. If MOPT = 1, return.

II. If MOPT = 2:

   A. For each compartment:

      1. Distribute the total effort based upon the proportion of the total target population that resided in the compartment at the beginning of the last timestep.

      2. Check to see if the effort ceiling has been exceeded:

         a. If no, continue.

         b. If yes, set effort = ceiling and accumulate excess effort.

   B. If excess effort ≤ 0, or if number of iteration > 10, return.

   C. If excess effort > 0 and number of iterations ≤ 10:

      1. Set total effort to be distributed = excess effort.

      2. Set total target population = the sum of the target populations in those compartments that have not reached their effort ceiling.

      3. Go back to (II.A) for next iteration.

   D. Return.

III. If MOPT = 3:

   A. Sum up the total target population in those compartments that have not yet reached their yield limit.

   B. Distribute the effort to those compartments that have not yet reached their yield limit based upon the fraction of the total target population that resided in each compartment at the beginning of the last timestep.

   C. Return.

Figure 9.--Subroutine DATIN outline.

I. Transfer input data to file 3.

II. Read model structure inputs.

III. Branch to appropriate management option inputs.

   A. If MOPT = 1: for each compartment, read the commercial fishing effort inputs.

   B. If MOPT = 2 or 3:

      1. Read total commercial fishing effort inputs.

      2. For each compartment, read ceiling inputs.

IV. For each compartment, read sport fishing effort inputs.

V. For each species:

   A. Read reproduction coefficient inputs.

   B. For each compartment, read commercial fishing catchability coefficient inputs.

   C. For each compartment, read sport fishing catchability coefficient inputs.

   D. For each compartment:

      1. Read number of compartments migrated to.

      2. For each compartment migrated to, read migration coefficient inputs.

Figure 10.--Subroutine READ4 outline.

I. Rewind file 3.

II. Branch to the appropriate management option inputs.

   A. If MOPT = 1: for each compartment, read the next four nodes of the explicitly represented commercial fishing effort inputs.

   B. If MOPT = 2 or 3:

      1. If total commercial fishing effort is explicitly represented, read the next four nodes.

      2. For each compartment, read the next four nodes of the explicitly represented ceiling inputs.

III. For each compartment, read the next four nodes of the explicitly represented sport fishing effort inputs.

IV. For each species:

   A. If the reproduction coefficient is explicitly represented, read the next four nodes.

   B. For each compartment, read the next four nodes of the explicitly represented commercial fishing catchability coefficient inputs.

   C. For each compartment, read the next four nodes of the explicitly represented sport fishing catchability coefficient inputs.

   D. For each compartment:

      1. Read number of compartments migrated to and type of representation for migration coefficients.

      2. If explicitly represented: for each compartment migrated to, read the next four nodes of the migration coefficient inputs.

N. Subroutine FIND

Subroutine FIND finds a specified card among the input cards for an explicitly represented variable, reads the card, and skips to the end of the card set for that variable.

O. Subroutine SKIP

Subroutine SKIP skips a specified number of cards on the input file.

VII. VARIABLE DEFINITION LIST

The following is an alphabetically arranged definition list of the major variables in the BFISH model.

B: The difference between the end-of-timestep and beginning-of timestep values of a function being evaluated in subroutine EVAL.

C (I, J, ICMPT, ISPEC):

1) If ICTYPE (I, ICMPT, ISPEC) = 1, C(I, J, ICMPT, ISPEC) holds the J^{th} polynomial coefficient, for species ISPEC in compartment ICMPT, for (if I = 1) the commercial fishing catchability coefficient or (if I = 2) the sport fishing catchability coefficient.

2) If ICTYPE (I, ICMPT, ISPEC) = 2, C(I, J, ICMPT, ISPEC) holds four nodal values (J = 3 to 6) from the input explicit representation, for species ISPEC in compartment ICMPT, for (if I = 1) the commercial fishing catchability coefficient or (if I = 2) the sport fishing catchability coefficient.

CARD: A character array (80 character capacity) used in subroutine DATIN to transfer input records from file 5 to file 3.

COEF: Used in subroutine SUMGRT to store the migration coefficient between two specified compartments at a specified time.

COEFS (I): Used in subroutines EVAL and PRIME to store the I^{th} coefficient of the specific polynomial function being evaluated.

DUMMY: An array used to transfer four nodal values of some explicitly represented variable between subroutine FIND and subroutine READ4.

E (I, J, ICMPT):

1) I = 1, MOPT = 1:

a) IETYPE (1, ICMPT) = 1: E(1, J, ICMPT) holds the J^{th} polynomial coefficient for the commercial fishing effort function in compartment ICMPT.

b) IETYPE (1, ICMPT) = 2: E(1, J, ICMPT) holds four nodal values (J = 3 to 6) from the input explicit representation for the commercial fishing effort in compartment ICMPT.

2) I = 1, MOPT = 2:

a) ICLTYP (ICMPT) = 1: E(1, J, ICMPT) holds the polynomial coefficients (J = 2 to 6) for the commercial fishing effort ceiling function in compartment ICMPT. The commercial fishing effort in compartment ICMPT for the present timestep is stored in E(1, 1, ICMPT).

b) ICLTYP (ICMPT) = 2: E(1, J, ICMPT) holds four nodal values (J = 3 to 6) from the input explicit representation for the commercial fishing effort ceiling in compartment ICMPT. The commercial fishing effort in compartment ICMPT for the present timestep is stored in E(1, 1, ICMPT).

3) I = 1, MOPT = 3: E(1, 2, ICMPT) holds the yield limit per cycle for managed species in compartment ICMPT. The commercial fishing effort in compartment ICMPT for the present timestep is stored in E(1, 1, ICMPT).

4) I = 2:

   a) IETYPE (2, ICMPT) = 1: E(2, J, ICMPT) holds the J^{th} polynomial coefficient for the sport fishing effort function in compartment ICMPT.

   b) IETYPE (2, ICMPT) = 2: E(2, J, ICMPT) holds four nodal values (J = 3 to 6) from the input explicit representation for the sport fishing effort in compartment ICMPT.

EC: Commercial fishing effort.

ECDOT: First time derivative of the commercial fishing effort function.

EM: The sum of the migration coefficients out of a specific compartment.

EMIGRT (ICMPT): The sum of the migration coefficients out of compartment ICMPT.

ES: Sport fishing effort.

ESDOT: First time derivative of the sport fishing effort function.

EXCESS: Used in subroutine EFFORT (when MOPT = 2) to store the excess effort accumulated during one iteration that must be distributed on the next iteration.

F1: The beginning-of-time-interval value of some function that is to be integrated.

F1DOT: The beginning-of-time-interval first derivative of some function that is to be integrated.

F2: The end-of-time-interval value of some function that is to be integrated.

F2DOT: The end-of-time-interval first derivative of some function that is to be integrated.

FP2: The estimate for the second time drrivative of some function, evaluated at the beginning of a time interval, for use in a Taylor series expansion of the function across the time interval.

FP3: The estimate for the third time derivative of some function, evaluated at the beginning of a time interval, for use in a Taylor series expansion of the function across the time interval.

FP4: The estimate for the fourth time derivative of some function, evaluated at the beginning of a time interval, for use in a Taylor series expansion of the function across the time interval.

FRAC (ICMPT): The fraction of the beginning-of-timestep population of a given species that resides in compartment ICMPT.

FRACP: The fraction of the total target population that resided in a given compartment at the beginning of the previous timestep.

H: Length of one timestep.

IBEGIN: The nodal array location containing the beginning-of-timestep value of an explicitly represented variable.

IC: Input card (record) number.

ICARD: The input card (record) number that contains the next four nodal values for the explicitly represented variables.

ICHECK: A check to see if the number of timesteps per cycle is a multiple of four.

ICLORD (ICMPT): The order of the polynomial representing the effort ceiling in compartment ICMPT (used only if ICLTYP (ICMPT) = 1).

ICLTYP (ICMPT): 1 or 2 for polynomial of explicit representation (respectively) of the effort ceiling function for compartment ICMPT.

ICMPT: Compartment number.

ICORDR (I, ICMPT, ISPEC):

1) I = 1: The order of the polynomial representing the commercial fishing catchability coefficient for species ISPEC in compartment ICMPT (used only if ICTYPE (1, ICMPT, ISPEC) = 1).

2) I = 2: The order of the polynomial representing the sport fishing catchability coefficient of species ISPEC in compartment ICMPT (used only if ICTYPE (2, ICMPT, ISPEC) = 1).

ICOUNT: An integer counter.

ICTYPE (I, ICMPT, ISPEC):

1) I = 1: 1 or 2 for polynomial or explicit representation, respectively, of the commercial catchability coefficient of species ISPEC in compartment ICMPT.

2) I = 2: 1 or 2 for polynomial or explicit representation, respectively, of the sport fishing catchability coefficient of species ISPEC in compartment ICMPT.

IEORDR (I, ICMPT):

1) I = 1: The order of the polynomial representing the commercial fishing effort in compartment ICMPT (used only if IETYPE (1, ICMPT) = 1).

2) I = 2: The order of the polynomial representing the sport fishing effort in compartment ICMPT (used only if IETYPE (2, ICMPT) = 1).

IETYPE (I, ICMPT):

1) I = 1: 1 or 2 for polynomial or explicit representation, respectively, of the commercial fishing effort in compartment ICMPT.

2) I = 2: 1 or 2 for polynomial or explicit representation, respectively, of the sport fishing effort in compartment ICMPT.

IF: Function index: 1 or 2 for commercial fishing or sport fishing functions, respectively.

IFIRST (ISPEC): The first timestep in the recruitment period for species ISPEC.

IFUNC: Function number where

1 = reproduction coefficient

2 = recruitment rate

3 = commercial fishing effort

4 = sport fishing effort

5 = commercial catchability coefficient

6 = sport catchability coefficient

7 = migration coefficient

8 = effort or yield ceiling

9 = total commercial fishing effort

IGO: Branching index.

IKEY: The number of target species in the target population for commercial fishing effort (used when MOPT = 2 or 3).

IKSPEC(I): Species number of the I^{th} species in the target population (I = 1 to IKEY).

ILAG: The lag time (in timesteps) between the target species distribution and the commercial fishing effort distribution (= 0 or 1).

ILAST (ISPEC): The last timestep in the recruitment period for species ISPEC.

IMIG: The migration coefficient option where

1) IMIG = 1, the migration coefficients are treated as continuous functions of time.

2) If IMIG = 2, the migration coefficients are treated as piecewise constant functions of time.

IMORDR (ICMPT, ITO, ISPEC): Order of the polynomial used to represent the migration coefficient for species ISPEC from compartment ICMPT to compartment MCMPT (ICMPT, ITO, ISPEC) (used only if IMTYPE (ICMPT, ISPEC) = 1).

IMTYPE (ICMPT, ISPEC): 1 or 2 for polynomial or explicit representation, respectively, of the migration coefficient(s) for species ISPEC leaving compartment ICMPT.

IQC: Commercial fishing catchability coefficient option, where

1) If IQC = 1, the commercial fishing catchability coefficients are treated as continuous functions of time.

2) If IQC = 2, the commercial fishing catchability coefficients are treated as piecewise constant functions of time.

IQS: Sport fishing catchability coefficient options, where

1) If IQS = 1, the sport fishing catchability coefficients are treated as continuous functions of time.

2) If IQS = 2, the sport fishing catchability coefficients are treated as piecewise constant functions of time.

IRCRUT (ISPEC): = 1 if the present timestep is part of the recruitment period for species ISPEC; = 0 otherwise.

IREAD: The timestep number before which the next four nodal values for explicitly represented variables must be read.

IRFLAG: = 0 if no variable is explicitly represented; = 1 otherwise.

IRORDR (I, ISPEC):

1) 1: The order of the polynomial describing the reproduction coefficient for species ISPEC (used only if IRTYPE (1, ISPEC) = 1).

2) I = 2: The order of the polynomial describing the recruitment rate for species ISPEC (will always = 4).

IRP1: A counter usually equal to the number of coefficients in a polynomial.

IRTYPE (I, ISPEC):

1) J = 1: 1 or 2 for polynomial or explicit representation, respectively, of the reproduction coefficient for species ISPEC.

2) I = 2: 1 always since the recruitment rate for species ISPEC is always generated by BFISH as a polynomial.

IS: An integer counter.

ISKIP: An integer argument equal to the number of cards (records) that are to be skipped on the input file.

ISPEC: Species number.

ISTEP: Timestep number.

ISTOP: Integer variable = MATURE (ISPEC).

IT: Integer variable indexing a compartment migrated to.

ITER: Iteration number.

ITO: Integer counter indexing a compartment migrated to.

ITORDR: The order of the polynomial representing the total commercial fishing effort (used only if ITTYPE = 1).

ITST: An array of timestep numbers.

ITTYPE: 1 or 2 for polynomial or explicit representation, respectively, of total commercial fishing effort.

IYKEY: Number of managed species in the managed population (used only when MOPT = 3).

IYSPEC (I): Species number of the Ith species in the managed population (I = 1 to IYKEY).

MATURE (ISPEC): Age (in cycles) at which members of species ISPEC enter the fishery.

MCMPT (ICMPT, ITO, ISPEC): Compartment numbers to which members of species ISPEC migrate from compartment ICMPT (ITO = 1, MTO (ICMPT, ISPEC))

MFLAG (ICMPT): = 1 if compartment ICMPT is in the management area; = 0 otherwise.

MOPT: Management option, where

MOPT	DESCRIPTION
1	Externally (user) supplied commercial fishing effort
2	Commercial fishing effort internally generated subject to user supplied effort ceilings
3	Commercial fishing effort internally generated subject to user supplied yield limits

MT (ICMPT, ISPEC): The number of different compartments migrated to from compartment ICMPT by species ISPEC.

NCARD: The number of input cards (records) necessary to explicitly represent each node in a time cycle. Hence, each explicitly represented variable (TYPE = 2) will have a set of NCARD input records associated with it.

NCMPT: The number of compartments being modeled in the present analysis.

NCYCLE: The number of time cycles being modeled in the present analysis.

NLAST: The array location holding the last node of the set of explicit nodal values currently in storage.

NMAX: The total number of timesteps that will be modeled during the present analysis.

NMCMPT: The number of managed compartments being modeled in the present analysis.

NORDER: Order of a polynomial being evaluated.

NSPEC: The number of species being modeled in the present analysis.

NSTEP: Number of timesteps per cycle.

NUMBER: An integer variable usually used to store the value of MTO (ICMPT, ISPEC).

P: Population.

PDOT: First time derivative of the population of a given species in a given compartment.

PDOTOT: The sum of PDOT over all compartments.

PMANAG: The sum of the populations of a given species over all managed compartments.

POP (ICMPT, ISPEC): Population of species ISPEC in compartment ICMPT.

PTARGT (ICMPT): The target population in compartment ICMPT.

PTOT (ISPEC): The population of species ISPEC summed over all compartments.

QC: Commercial fishing catchability coefficient.

QCDOT: First time derivative of the commercial fishing catchability coefficient.

QS: Sport fishing catchability coefficient.

QSDOT: First time derivative of the sport fishing catchability coefficient.

R (I, J, ISPEC):

1) I = 1: R(1, J, ISPEC) holds either the polynomial coefficients (in J = 1 to 6) or nodal values (in J = 3 to 6), for IRTYPE (1, ISPEC) = 1 or 2 respectively, for the reproduction coefficient for species ISPEC.

2) I = 2: R(2, J, ISPEC) holds the polynomial coefficients (J = 1 to 5) for the 4th order polynomial describing the recruitment rate to species ISPEC.

R1: Beginning of timestep value of the reproduction function.

R1DOT: Beginning of timestep value of the first time derivative of the reproduction function.

RBAR (ISPEC): The integral of the reproduction function for species ISPEC over the recruitment period.

RC: Recruitment rate.

RCHECK (ISPEC): The maximum number of fish that can be recruited to species ISPEC in one timestep (= RCMAX (ISPEC) * timestep length).

RCINIT (ISPEC): The initial recruitment rate to species ISPEC, used for the first MATURE (ISPEC) time cycles.

RCMAX (ISPEC): The maximum recruitment rate for species ISPEC.

RDSAVE (ISPEC): Beginning of recruitment period value of the first time derivative of the reproduction function for species ISPEC.

RINTGL: The integrated recruitment rate over the present timestep.

RP: Reproduction coefficient.

RPDOT: First time derivative of the reproduction function.

RSAVE (ISPEC): Beginning of recruitment period value of the reproduction function for species ISPEC.

RSTACK (J, K, ISPEC): The stack of values used to calculate the recruitment coefficients for species ISPEC.

K (= 1 to MATURE (ISPEC)) = the number of cycles previous to the current cycle that the stored values represent.

J (= 1 to 5) =

J	DESCRIPTION
1	$f(t_1)$
2	$\dot{f}(t_1)$
3	$f(t_2)$
4	$\dot{f}(t_2)$
5	$\int_{t_1}^{t_2} f\,dt$

where

$f(t) = RP(T) * P_{TOTAL}(t)$

$RP(t)$ = reproduction coefficient for species ISPEC at time t

$P_{TOTAL}(t)$ = total population (summed over all compartments) for species ISPEC at time t

$\dot{f}(t) = \frac{df}{dt}$ evaluated at time t

The recruitment coefficients for species ISPEC in any cycle are calculated from a Taylor series expansion whose parameters are calculated from the 5 stored values RSTACK (J, MATURE(ISPEC), ISPEC), J = 1 to 5.

RTOT: Total recruitment rate for a specific species.

S(ICMPT): The Collatz sensitivity coefficient for the Runge-Kutta calculations for compartment ICMPT for a specific species.

SMIGRT (ICMPT): For a specific species, the sum of the products of the migration coefficients into compartment ICMPT and the populations of the contributing compartments, or

$$\text{SMIGRT}(i) = \sum_{i \neq j} XM_{j \to i} * P_j$$

$XM_{j \to i}$ = migration coefficient from compartment j to compartment i for a specified species and time.

P_j = population in compartment j of a specified species at a specified time.

STAR: An alphabetic variable (= *) used to denote the end of the input file.

SUM: = SMIGRT (ICMPT), used as a function subroutine argument.

T: Time.

TE(J):

1) If ITTYPE = 1, TE(J) holds the J^{th} polynomial coefficient for the total commercial fishing effort function.

2) If ITTYPE = 2, TE(J) holds four nodal values (J = 3 to 6) from the input explicit representation of the total commercial fishing effort.

TEFORT: Total commercial fishing effort.

TIME(J):

1) J = 1: beginning of timestep time

2) J = 2: middle of timestep time

3) J = 3: end of timestep time

TT: The total target fish population summed over compartments that have not yet reached their effort ceiling limit.

TTARGT: The total target fish population.

TTOTAL: Total time expired.

XK1 (ICMPT): First Runge-Kutta coefficient for compartment ICMPT and a specific species.

XK2 (ICMPT): Second Runge-Kutta coefficient for compartment ICMPT and a specific species.

XK3 (ICMPT): Third Runge-Kutta coefficient for compartment ICMPT and a specific species.

XK4 (ICMPT): Fourth Runge-Kutta coefficient for compartment ICMPT and a specific species.

XM (J, ICMPT, ITO, ISPEC):

1) If IMTYPE (ICMPT, ISPEC) = 1, XM (J, ICMPT, ITO, ISPEC) holds the J^{th} polynomial coefficient for the migration coefficient for migration of species ISPEC from compartment ICMPT to compartment MCMPT (ICMPT, ITO, ISPEC).

2) If IMTYPE (J, ICMPT, ITO, ISPEC) = 2, XM (J, ICMPT, ITO ISPEC) holds from nodal vales (J = 3 to 6) from the input explicit representation of the migration coefficient for migration of species ISPEC from compartment ICMPT to compartment MCMPT (ICMPT, ITO, ISPEC).

XNMORT (ISPEC): Natural mortality coefficient for species ISPEC.

XPOLY: Storage variable for a partially evaluated polynomial used in function subroutine POLY.

Y1: Used in subroutine EVAL to denote the beginning-of-timestep value of a function.

Y2: Used in subroutine EVAL to denote the end-of-timestep value of a function.

YC1 (ICMPT): The commercial yield function for compartment ICMPT evaluated at the beginning of the timestep.

YC1DOT (ICMPT): The first time derivative of the commercial yield function for compartment ICMPT evaluated at the beginning of the timestep.

YCMNG: Commercial fishing yield in the management area over a one timestep period.

YCMTOT (ISPEC): Cycle-to-date commercial fishing yield in the management area of species ISPEC.

YCSTEP (ISPEC): Total commercial fishing yield (summed over all compartments) of species ISPEC over a one timestep period.

YCTOT (ISPEC): Total cycle-to-date commercial fishing yield (summed over all compartments) of species ISPEC.

YIELDC: Commercial fishing yield.

YIELDS: Sport fishing yield.

YMANAG (ICMPT): Cycle-to-date yield of managed species in compartment ICMPT.

YS1 (ICMPT): The sport yield function for compartment ICMPT evaluated at the beginning of the timestep.

YS1DOT (ICMPT): The first time derivative of the sport yield function for compartment ICMPT evaluated at the beginning of the timestep.

YSMNG: The sport fishing yield in the management area over a one timestep period.

YSMTOT (ISPEC): Cycle-to-date sport fishing yield in the management area of species ISPEC.

YSSTEP (ISPEC): Total sport fishing yield (summed over all compartments) of species ISPEC over a one timestep period.

YSTOT (ISPEC): Total cycle-to-date sport fishing yield (summed over all compartments) of species ISPEC.

Z: Multiplier (= 0., 0.5, or 1.) used in subroutine EVAL when evaluating explicitly represented functions.

VIII. NOTES

A. BFISH backspaces, rewinds, and rereads the input file repeatedly during execution. This may cause laughter in the control room if the input unit happens to be a card reader. To avoid this, BFISH writes the entire input file to disc before starting its reads. To facilitate this, the user must make a disc file with logical unit number 3 available to the program before the execution step.

B. Increasing the number of species and compartments that may be modeled is as simple as increasing the dimensions of the appropriate arrays in the common blocks.

C. If you elect to use polynomials to represent any of the time dependent coefficients, be sure that you know what you are doing. At no time should any of the coefficients be negative. The BFISH program will be quite happy with negative coefficients, but the user may not be as content with the results of the analysis if fish migrate backwards or fishing fleets supplement the population by throwing fish into the water.

This non-negativity criterion holds, of course, for explicit representation (type 2) as well. But in this case the user can easily inspect the inputs for negative values.

A good personal rule would be to inspect a plot of each of your input polynomials before using it.

D. When the maximum recruitment rate ceiling, RCMAX, is imposed or lifted in the middle of a timestep, the Taylor series representation will try to match the resulting boundary conditions with a smooth function. This can lead to wild representations between time nodes. When the necessary information is then passed to subroutine RSOLVE to generate a recruitment function, a wildly oscillating function may result. This will manifest itself in unreasonable recruitment rates. Due to the nature of the calculations the end-of-cycle totals are usually realistic (the integrated functions are reasonable), but the interim timestep values for the recruitment rate may be negative, exceed RCMAX, or act in other suspicious ways. One example of how this condition may occur would be the combination of a large population and an end-of-timestep reproduction coefficient of zero. Thus, RCMAX will be imposed over most of the timestep, but eventually (as the reproduction function approaches zero) the RCMAX barrier will be broken and a large negative gradient will immediately ensue. This radical discontinuity would overtax a third order Taylor series. It is better to keep the reproduction coefficient positive and finite over the entire span of the recruitment period.

Note also that this will not manifest itself in the Collatz coefficient S. In fact, if the population gradient is purely a function of time, XK2 = XK3 and S = 0. The Runge-Kutta procedure is, in fact, probably performing admirably. It is the integrand that is unrealistic.

E. While it is not necessary, it would certainly be intuitively pleasing if the end-of-cycle coefficient values equal the beginning-of-cycle values ($f(t_F) = f(0)$). This is done by BFISH for explicit variables, but is up to the user for polynomials.